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On the determination of the optical-model potential in terms of the diffraction model parameters

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Abstract. The determination of the optical-model potential for a high-energy charged particle using the diffraction model phase shifts is discussed. It is shown that the finiteness of the nuclear charge distribution hitherto neglected makes a non-negligible contribution to the real part of the calculated potential at small distances. It is further observed that the form of the calculated potential is very much dependent on the model used for parametrizing the phase shifts.

1. Introduction

In a recent note Abul-Magd *et al* (1971) suggested that the nuclear phase shifts as determined from the diffraction model analysis and the high-energy relation between the phase shift and the potential (Glauber 1959) may be used to determine the optical-model potential for highly absorbing projectiles at intermediate and high energies. Using this approach these authors calculated the optical potential for the scattering of 104 MeV α particles from ^{208}Pb and ^{64}Ni. Earlier Elton (1966) also followed the same approach to investigate the qualitative features of the proton optical potential at 180 MeV. In these works we, however, find that due consideration of the finiteness of the nuclear charge distribution has not been given. Either it has been neglected or its effect on the calculated optical potential was tacitly assumed to be negligible. Our aim in this paper is to show that the finiteness of the nuclear charge distribution has, in general, non-negligible effects on the calculated real part of the optical potential at small nuclear interaction distances. We further observe that the calculated potential is very much dependent on the model used for parametrizing the phase shifts.

2. Theoretical consideration

The elastic scattering amplitude $f(\theta)$ describing the scattering of a charged nuclear particle from a target nucleus may be written as

$$f(\theta) = \frac{\mathrm{i}}{2k} \sum_{l=0}^{\infty} (2l+1)\{1-\exp[2\mathrm{i}(\sigma_l+\delta_l)]\}P_l(\cos\theta), \tag{2.1}$$

where k is the wavenumber, σ_l is the point-charge Coulomb phase shift and δ_l is the

phase shift due to the nuclear potential plus the difference between the point-charge and the extended-charge Coulomb potential.

The diffraction model approach to the scattering of a highly absorbing projectile consists of replacing the 'nuclear' scattering function $S_l = \exp(2i\delta_l)$ by some smooth function of l having few parameters which are determined by making a least-squares fit to the data. The parametrization is such that it incorporates those features of S_l on which the scattering amplitude depends crucially.

At sufficiently high energies when the wavelength is small compared to the interaction radius R and small-angle scattering dominates, the sum over l in equation (2.1) may be replaced approximately by an integration over the impact parameter $b \simeq l/k$ giving (Rodberg and Thaler 1967):

$$f(\theta) = ik \int_0^\infty db b J_0(qb)[1 - \exp(i\chi_t(b))], \tag{2.2}$$

where q is the momentum transfer and the total phase shift function $\chi_t(b)$ is defined as follows:

$$\chi_t(b) = \chi_{PC}(b) + \chi_N(b) \tag{2.3a}$$

$$\simeq (2\sigma_l + 2\delta_l)|_{l \simeq kb}. \tag{2.3b}$$

Since for highly absorbing projectiles the Glauber approximation is found to be fairly good even at the intermediate energies (Simbel and Abul-Magd 1970), Abul-Magd *et al* (1971) suggested that the phase shift function $\chi_N(b) = 2\delta_{l \simeq kb}$ as determined from the diffraction model analyses and the relation (Glauber 1959)

$$V(r) = \frac{\hbar v}{\pi} \frac{1}{r} \frac{d}{dr} \int_r^\infty \frac{\chi(b) b \, db}{(b^2 - r^2)^{1/2}} \tag{2.4}$$

may be used to determine the optical-model potential for the scattering of 104 MeV α particles. In equation (2.4) $V(r)$ is the potential leading to the phase shift function $\chi(b)$ relative to the free field.

However, we observe that when the point-charge Coulomb field is modified due to the extended nature of the nuclear charge distribution, as the case for the scattering of protons and α particles from nuclei is, the potential calculated by substituting $\chi_N(b)$ $(= 2\delta_l)$ for $\chi(b)$ in equation (2.4), strictly speaking, is not identifiable with the conventional optical-model potential. This is because, as already described, the so called nuclear phase shift δ_l is not only due to the nuclear interaction approximated by a complex potential but also to the modification in the point-charge Coulomb field caused by the extended nuclear charge distribution. Furthermore $\chi(b)$ occurring in equation (2.4) refers to a phase shift function relative to the free field whereas $\delta_l(b)$ is the phase shift (and hence the phase shift function $\chi_N(b)$) relative to the point-charge Coulomb field.

When the finiteness of the nuclear charge distribution is non-negligible the correct procedure, within the framework of the proposed method, is to use, instead of $\chi_N(b)$ the total phase-shift function $\chi_t(b) = \chi_N(b) + \chi_{PC}(b)$, in equation (2.4), to obtain for the

total potential $V(r)$ the expression

$$V(r) = V_N(r) + V_{PC}(r) \tag{2.5}$$

where

$$V_i(r) = \frac{\hbar v}{\pi}\frac{1}{r}\int_0^\infty \frac{d}{dr}\chi_i(t^2+r^2)^{1/2}\,dt, \qquad (i = N, PC). \tag{2.6}$$

The first term in equation (2.5) is the same as that calculated by Abul-Magd *et al* (1971) and identified as the optical-model potential. The second term can be easily evaluated by writing

$$V_{PC}(r) = \frac{\hbar v}{\pi}\int_0^\infty dt\left[\frac{1}{b}\frac{d\chi_{PC}}{db}\right]_{b=(t^2+r^2)^{1/2}} \tag{2.7}$$

and making use of the relation for large l (Rodberg and Thaler 1967):

$$\Delta\sigma = \frac{ZZ'e^2}{\hbar v}\frac{\Delta l}{l}$$

(where Z, Z' are the atomic numbers of the target nucleus and the incident particle respectively) which in terms of the impact parameter b reads

$$\frac{d\chi_{PC}}{db} = \frac{2ZZ'e^2}{\hbar v}\frac{1}{b}.$$

The result is $V_{PC}(r) = ZZ'e^2/r$ as expected. Alternatively, one may use the high-energy expression for the point-charge Coulomb phase-shift function:

$$\chi_{PC}(b) = \frac{2ZZ'e^2}{\hbar v}\ln\left(\frac{b}{2a}\right),$$

where a is the arbitrarily large screening radius (Glauber 1959), to obtain the same result (screening has no observable effect on the scattering).

The potential $V(r)$ should be identified with the total potential $V_{OP}(r) + V_C(r)$ used in the optical-model calculations for charged projectiles, where $V_{OP}(r)$ and $V_C(r)$ are the complex optical-model and the modified Coulomb potentials respectively. This leads to the expression

$$V_{OP}(r) = V_N(r) + \left(\frac{ZZ'e^2}{r} - V_C(r)\right), \tag{2.8}$$

which differs from that calculated earlier (here denoted by $V_N(r)$) by the term within the large parentheses. Therefore, it follows that unless $V_C(r) = ZZ'e^2/r$, the real part of the potential in earlier calculations is not identifiable with the corresponding part of the optical-model potential, clearly Im $V_{OP}(r)$ = Im $V_N(r)$.

Using equation (2.6) we may express the real and imaginary parts of $V_N(r)$ in terms of the 'nuclear' scattering function $S_l = \exp(2i\delta_l)$ after the substitution $l \simeq kb$. The

resulting relations are:

$$\text{Re } V_N(r) = \frac{\hbar v}{\pi}\int_0^\infty \left[\frac{1}{b}\frac{\mathrm{d}(\arg S)}{\mathrm{d}b}\right]_{b=(t^2+r^2)^{1/2}} \mathrm{d}t \tag{2.9a}$$

and

$$\text{Im } V_N(r) = -\frac{\hbar v}{\pi}\int_0^\infty \left[\frac{1}{|S|}\frac{\mathrm{d}|S|}{\mathrm{d}b}\right]_{b=(t^2+r^2)^{1/2}} \mathrm{d}t \tag{2.9b}$$

which we have used in § 3.

3. Calculation and discussion

In the following we apply equation (2.8) to calculate the optical-model potential for the scattering of 104 MeV α particles from nuclei. The nuclear part $V_N(r)$ is calculated using equations (2.9) and the generalized parametrization

$$|S_l| = \epsilon + \frac{1-\epsilon}{1+\exp[(L-l)/\delta_1]} \tag{3.1a}$$

$$\arg S_l = [a_0 + a_1(l-L) + \ldots]\{1+\exp[(l-L)/\delta_2]\}^{-1} \tag{3.1b}$$

employed by Hauser *et al* (1969) to obtain very good fits to the elastic angular distribution data. To calculate $V_C(r)$ we use, following Igo and Thaler (1957),

$$V_C(r) = \begin{cases} \dfrac{ZZ'e^2}{R}\left[\dfrac{1}{n^2}+\dfrac{1}{2}-\dfrac{x^2}{6}+\dfrac{e^{-n}}{n^2}\left(\dfrac{1-e^{nx}}{nx}+\dfrac{e^{nx}}{2}\right)\right]\left(\dfrac{1}{3}+\dfrac{2}{n^2}+\dfrac{e^{-n}}{n^3}\right)^{-1}, & x \leqslant 1 \\ \dfrac{ZZ'e^2}{R}\left[\dfrac{1}{x}-e^{n(1-x)}\left(\dfrac{1}{x}+\dfrac{n}{2}\right)\left(e^{-n}+2n+\dfrac{n^3}{3}\right)^{-1}\right], & x > 1 \end{cases} \tag{3.2}$$

where $x = r/R$, $R = 1{\cdot}3A^{1/3}$ and n is a parameter to be discussed shortly. This expression is obtained by assuming $V_C(r)$ to be that from a fictitious radial charge distribution which takes into account approximately the finite extent of the charge distributions of both the bombarding and target nucleus and is of the form (Hill and Ford 1954, Igo and Thaler 1957)

$$\rho(x) = \frac{\rho_0}{(1-\frac{1}{2}e^{-n})}\begin{cases} 1-\frac{1}{2}e^{-n(1-x)}, & x \leqslant 1 \\ \frac{1}{2}e^{-n(x-1)}, & x > 1 \end{cases} \tag{3.3}$$

where ρ_0 describes the central charge density and $1 \leqslant n < \infty$. The diffuseness of the fictitious charge distribution is described by the parameter n which is related to the 90% to 10% fall of distance t through $n = 2R\ln 5/t$. Since for the commonly used Saxon–Wood charge distribution t is related to the diffuseness parameter a through $t = 4a\ln 3$, the predictions of the charge density given by equation (3.3) are essentially the same as those of a Saxon–Wood distribution of the half density radius R and $a = 0{\cdot}73R/n$. (For all practical purposes the term $\frac{1}{2}e^{-n}$ in the denominator in equation (3.2) is negligible for not very light nuclei.) Further it may be verified that in the limit of

large n equations (3.3) and (3.2) give, respectively, the uniform charge distribution and the corresponding Coulomb potential $V_C(r)$.

In this work we have chosen rather arbitrarily $t = 3{\cdot}5$ fm. (A slightly large value is taken in order to simulate the finiteness of the charge distribution of both the projectile and the target.) Physically reasonable variation about this value of t in the case of the distribution employed here or use of any other physically acceptable form for $\rho(r)$ would in no way affect the qualitative conclusions of the present work.

The results of the calculation for a moderately heavy nucleus (^{124}Sn) are presented in figure 1. It is seen that the finiteness of the nuclear charge distribution has non-negligible effects at small nuclear interaction distances. For heavier nuclei the effect is expected to be larger.

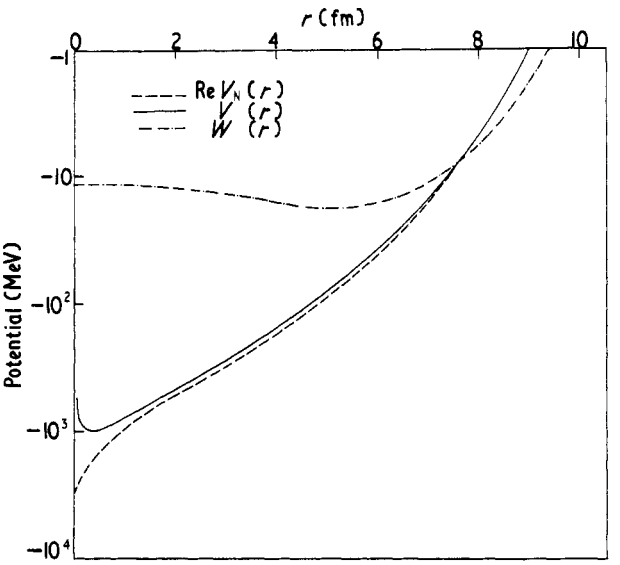

Figure 1. The calculated optical potential for α–^{124}Sn scattering using the diffraction-model phase shifts of Hauser *et al* (1969). The full and broken curves respectively represent the real part of the potential with and without including the finiteness of the nuclear charge distribution.

Using the parametrization (3.1) and the parameter values as given by Hauser *et al* (1969) we have also calculated $V_N(r)$ for ^{208}Pb. The forms of the calculated real (imaginary) parts of $V_N(r)$ turn out to be the same as shown in figure 1. Recalling the calculations by Abul-Magd *et al* (1971) using the parametrization of Ericson (1966), this implies that the calculated potential depends very much on the model used for parametrizing the phase shifts. In fact it is amusing to note that the shape of the real (imaginary) $V_N(r)$ resulting from the generalized parametrization resembles that of the imaginary (real) $V_N(r)$ as calculated by Abul-Magd *et al* (1971). It thus appears unlikely that the proposed method would be helpful in resolving the ambiguities in the conventional optical-model potential.

Further, the finite charge distribution correction when applied to the calculation of Abul-Magd *et al* (1971) would manifest itself much more pronouncedly than shown

here. Because of small negative values of Re $V_N(r)$ within a large interior of the interaction region (figure 3 of Abul-Magd *et al* 1971) the inclusion of the correction in this case would cause Re $V_N(r)$ to change sign just inside the surface region and finally make it highly repulsive around the origin.

It may be remarked finally that since, in the presence of a large absorption at the nuclear surface, the scattering is mainly governed by the behaviour of the optical potential in the surface region (Igo and Thaler 1957), the finite charge distribution correction as shown in figure 1 is unlikely to have any appreciable effect on the calculation of the cross sections. However, this may not generally be the case. It may happen that the calculated Re $V_N(r)$ turns out to be quite small compared to the correction term over a large part of the interaction region (an example of this kind has already been given). If so, the correction is expected to affect the calculation of the cross sections as well. Whatever is the case the present work clearly demonstrates the importance of considering the finiteness of the nuclear charge distribution for drawing any realistic conclusion concerning the radial distribution of the real potential predicted by the diffraction model phase shifts.

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